

## Meson Condensation in Dense Matter Revisited\*

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### Abstract

The results for meson condensation in the literature vary markedly depending on whether one uses chiral perturbation theory or the current-algebra-plus-PCAC approach. To elucidate the origin of this discrepancy, we re-examine the role of the sigma-term in meson condensation. We find that the resolution of the existing discrepancy requires a knowledge of terms in the Lagrangian that are higher order in density than hitherto considered.

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Kaon condensation in dense nuclear matter was proposed some time ago by Kaplan and Nelson [1], who used a particular effective Lagrangian derived from chiral perturbation theory (ChPT) and employed the tree approximation. In this Lagrangian the attractive force that drives condensation is provided primarily by the  $K$ - $N$  sigma term, which is expected to be much larger than the  $\pi$ - $N$  sigma term. For  $\Sigma_{KN} = (400 \sim 600)$  MeV, the critical density for kaon condensation was predicted to be  $\rho_c = 2 \sim 3 \rho_0$  ( $\rho_0$  = normal nuclear density) [1]. This remarkable result gave strong impetus to further detailed studies of kaon condensation and its possible influences on neutron stars [2-8]. Along the line of ChPT, a systematic examination of higher order terms in chiral expansion has been pursued using the heavy-fermion formalism [5,7,8]. Meanwhile, several authors [9-13] have recently questioned the validity of kaon condensation driven by the  $K$ - $N$  sigma term. In particular, Yabu et al. [11] demonstrated explicitly that the use of  $K$ - $N$  scattering amplitudes that respect the current algebra theorems and PCAC does not lead to kaon condensation. An important question is why the *existing* calculations on kaon condensation give markedly different results depending on whether one uses ChPT or the current algebra approach. In this note we analyze the nature of the problem involved and discuss what kind of additional information is required to settle the issue. Since the sigma term is the central issue here, we first concentrate our attention on the role of the sigma term. Furthermore, for the illustrative purpose, we consider s-wave pion condensation rather than kaon condensation itself. We will argue that terms of  $\mathcal{O}(\rho^2)$  in the Lagrangian are of importance to resolve the meson condensation problem. After addressing this main point, we also discuss the relation of our argument with the latest detailed calculation by Lee et al. [8] that includes up to one-loop diagrams in ChPT.

We first describe the essential feature of the original treatment of s-wave meson condensation based on the sigma term of an effective Lagrangian [1]. As a toy model we use the lowest-order ChPT expansion containing s-wave  $\pi$ -nucleon interaction and further truncate this Lagrangian to the minimum number of terms in order to illustrate our points:

$$\mathcal{L}_1 = \frac{1}{2} \left[ -\phi(\square + m_\pi^2)\phi + \frac{\Sigma_{\pi N}}{f_\pi^2} \phi^2 \bar{N}N \right], \quad (1)$$

where  $\phi(x)$  and  $N(x)$  are the pion and the nucleon field, respectively, and  $f_\pi$  is the pion decay constant. The  $\Sigma_{\pi N}$  is the  $\pi$ - $N$  sigma term,

$$\Sigma_{\pi N} = \frac{1}{2}(m_u + m_d) \langle N | \bar{u}u + \bar{d}d | N \rangle. \quad (2)$$

For  $\mathcal{L}_1$ , the  $\pi$ - $N$  scattering amplitude in tree approximation is given by

$$T_{\pi N}^{(1)} = \frac{\Sigma_{\pi N}}{f_\pi^2}. \quad (3)$$

To estimate the effective pion mass  $m_\pi^*$  in nuclear matter, we may use the mean-field approximation and replace the nucleon operator  $\bar{N}N$  in (1) with the nuclear matter density  $\rho$ . Then the pion dispersion relation becomes  $\omega^2 - \mathbf{k}^2 - m_\pi^2 + \rho \cdot \Sigma_{\pi N}/f_\pi^2 = 0$ . The effective pion mass  $m_\pi^*$  is defined by  $m_\pi^* \equiv \omega(\mathbf{k} = 0)$ , and the critical density  $\rho_c$  for pion condensation is determined from the condition  $m_\pi^* = 0$ . In the present case we obtain

$$[m_\pi^*(1)]^2 = m_\pi^2 - \rho \frac{\Sigma_{\pi N}}{f_\pi^2}. \quad (4)$$

and

$$\rho_c = \frac{m_\pi^2 f_\pi^2}{\Sigma_{\pi N}}. \quad (5)$$

The second approach used in [10, 11] may be summarized as follows. One defines the pion extrapolating field  $\pi(x)$  by

$$\pi(x) \equiv \frac{1}{m_\pi^2 f_\pi} \partial_\mu A^\mu(x) = \frac{m_q}{m_\pi^2 f_\pi} \bar{q}(x) \gamma_5 q(x), \quad (6)$$

where  $A^\mu(x)$  is the axial current, and the last equality is given by QCD. Due to its “simplicity” this definition of the pion field is frequently used in QCD, the NJL model and the non-linear sigma model  $\pi^a(x) = \text{Tr}(\tau^a U(x))$ . With this operator  $\pi(x)$ , the  $\pi$ - $N$  scattering amplitudes for on- and off-shell momenta of the pions are “defined” by

$$T_{\pi N}^{(2)} = i^2 (m_\pi^2 - (k')^2)(m_\pi^2 - k^2) \int d^4x d^4y e^{ik'x} e^{-iky} \langle N' | T \pi(x) \pi(y) | N \rangle, \quad (7)$$

where  $k$  ( $k'$ ) is the incoming (outgoing) pion momentum. The amplitude  $T_{\pi N}$  in (7) satisfies the Adler condition and, at the Weinberg point, it also satisfies the well-known relation with the sigma term [14, 15]. For forward scattering, the general form of  $T_{\pi N}$  that is consistent with the low-energy theorems can be written as

$$T_{\pi N}^{(2)} = \frac{k^2 + (k')^2 - m_\pi^2}{f_\pi^2 m_\pi^2} \Sigma_{\pi N} + T'_{\pi N}, \quad (8)$$

where only the  $\Sigma_{\pi N}$ -dependent terms are explicitly shown; these terms become identical to the amplitude in eq.(3) for on-mass-shell mesons [17]. The remaining term,  $T'_{\pi N}$ , contains

the Born terms, the Weinberg-Tomozawa term, etc., and gives important contributions to the on-shell  $\pi$ - $N$  scattering amplitude [9, 10, 11, 12, 18]. However, here we neglect these terms in order to concentrate on the role of the sigma term. Applying the mean field approximation, the  $m_\pi^*$  that corresponds to the  $\pi$ - $N$  amplitude eq.(8) is found to be

$$[m_\pi^*(2)]^2 = m_\pi^2 \frac{1 + \rho \frac{\Sigma_{\pi N}}{m_\pi^2 f_\pi^2}}{1 + 2\rho \frac{\Sigma_{\pi N}}{m_\pi^2 f_\pi^2}}. \quad (9)$$

At very low nuclear densities  $m_\pi^*(1) \approx m_\pi^*(2)$  but, for larger  $\rho$ ,  $m_\pi^*(1)$  and  $m_\pi^*(2)$  behave very differently. In particular, eq.(9) tends to  $m_\pi/\sqrt{2}$  as  $\rho \frac{\Sigma_{\pi N}}{m_\pi^2 f_\pi^2} \rightarrow \infty$ , rendering meson condensation highly unlikely.

The difference between  $m_\pi^*$  of eq.(9) and  $m_\pi^*$  of eq.(4) represents the gist of the current controversy on meson condensation. In view of the great phenomenological success of ChPT and the PCAC approaches, it is puzzling that their predictions on  $m_\pi^*$ , as they stand, differ so drastically [20]. In what follows we clarify the origin of this discrepancy and show that  $\mathcal{O}(\rho^2)$  terms are necessary to resolve the problem.

Before going into the specificity, we first recall a general argument. For a given Lagrangian  $\mathcal{L}$ , the finite-density pion Green function is defined by

$$G_\rho(x; \varphi) = \langle \rho | T \varphi(x) \varphi(0) | \rho \rangle, \quad (10)$$

where  $|\rho\rangle$  is the ground state (with baryon density  $\rho$ ) of the system governed by  $\mathcal{L}$ , and  $\varphi(x)$  is an arbitrary operator for the pion field. The field  $\varphi$  can be anything so long as it connects one-pion state to vacuum, i.e.,  $\langle \pi | \varphi(x) | 0 \rangle \neq 0$ . The pole position of  $G_\rho(x; \varphi)$  corresponds to the energy  $E_n$  of a pionic-mode intermediate state  $|n\rangle$  that can be connected to  $|\rho\rangle$  via  $\varphi$ . Note that  $E_n$ , which is determined by  $\mathcal{L}$  itself, is *independent* of the choice of  $\varphi$ . It then follows that  $m_\pi^*$ , which is uniquely given by the pole position of  $G_\rho(x; \varphi)$ , must be independent of  $\varphi$ .

Now, the two amplitudes  $T_{\pi N}^{(1)}$  eq.(3) and  $T_{\pi N}^{(2)}$  eq.(8), although identical on the mass shell, exhibit completely different off-mass-shell behaviors. As is well known, the off-mass-shell values of the  $\pi$ - $N$  amplitudes depend on the choice of the extrapolating field. In our case the difference between  $T_{\pi N}^{(1)}$  and  $T_{\pi N}^{(2)}$  reflects the two non-equivalent extrapolating fields,  $\phi(x)$  [eq.(1)] and  $\pi(x)$  [eq.(6)]. Naively, one might ascribe the variance between  $m_\pi^*(1)$  and  $m_\pi^*(2)$  to the different off-mass-shell behaviors of the  $\pi$ - $N$  scattering amplitudes. This interpretation, however, is invalidated by the above general argument; even if  $\pi$ - $N$  scattering amplitudes exhibit different off-mass-shell behaviors corresponding to different

extrapolation fields,  $m_\pi^*$  itself should remain unaffected insofar as the Lagrangian of the system is held fixed. Therefore, it is not appropriate to attribute the discrepancy between  $m_\pi^*(1)$  and  $m_\pi^*(2)$  to the off-mass-shell problem.

To gain more insight into the nature of this difference, we consider an effective Lagrangian  $\mathcal{L}_2$  which, at the tree level, reproduces the first term of the  $\pi$ - $N$  scattering amplitude (8) and leads to the effective mass eq.(9):

$$\mathcal{L}_2 = \frac{1}{2} \left[ -\pi(\Box + m_\pi^2)\pi - \frac{\Sigma_{\pi N}}{f_\pi^2}(\pi^2 + \frac{2}{m_\pi^2}\pi\Box\pi)\bar{N}N \right]. \quad (11)$$

$\mathcal{L}_2$  differs from  $\mathcal{L}_1$  [eq.(1)] by the existence of the interaction term that involves  $\Box\pi$  (“box term”) [23]. Since the meson field in ChPT is nothing more than an integration variable and has no physical meaning by itself [19, 24], it is useful to examine here to what extent  $\mathcal{L}_2$  can be transformed into  $\mathcal{L}_1$  via a meson field redefinition. To this end, we apply the mean field approximation,  $\bar{N}N \rightarrow \rho$ , to (11) [25] and introduce a new meson field  $\tilde{\phi}(x)$  defined by

$$\pi(x) = \left( 1 - \rho \frac{\Sigma_{\pi N}}{f_\pi^2 m_\pi^2} \right) \tilde{\phi}(x). \quad (12)$$

With  $\tilde{\phi}(x)$ , the Lagrangian (11) can be rewritten as

$$\mathcal{L}_2 = \left( 1 - \rho \frac{\Sigma_{\pi N}}{f_\pi^2 m_\pi^2} \right)^2 \frac{1}{2} \left[ -\tilde{\phi}(\Box + m_\pi^2)\tilde{\phi} - \frac{\Sigma_{\pi N}}{f_\pi^2}(\tilde{\phi}^2 + \frac{2}{m_\pi^2}\tilde{\phi}\Box\tilde{\phi})\rho \right]. \quad (13)$$

Expanding this in  $\rho$ , we obtain

$$\mathcal{L}_2 = \frac{1}{2} \left[ -\tilde{\phi}(\Box + m_\pi^2)\tilde{\phi} + \rho \frac{\Sigma_{\pi N}}{f_\pi^2} \tilde{\phi}^2 \right] + \mathcal{O}(\rho^2). \quad (14)$$

This Lagrangian is identical to eq.(1) ( $\phi \leftrightarrow \tilde{\phi}$ ), if the terms of  $\mathcal{O}(\rho^2)$  are neglected, a feature which is in accord with the fact that  $m_\pi^*(1) = m_\pi^*(2)$  up to order  $\rho$ .

The equivalence of  $\mathcal{L}_1$  and  $\mathcal{L}_2$  breaks down at the  $\mathcal{O}(\rho^2)$  level, and this non-equivalence is responsible for the difference between  $m_\pi^*(1)$  and  $m_\pi^*(2)$ . Although this statement itself is correct, the real significance of this statement hinges upon the question: Can the existing formalisms make a meaningful distinction between  $m_\pi^*(1)$  and  $m_\pi^*(2)$ ? For the sake of clarity, we rephrase this question by referring back to the general discussion of  $G_\rho(x; \varphi)$  [eq.(10)]. For the Lagrangian  $\mathcal{L}_2$ , one can consider two Green functions,  $G^{(2)}(x; \pi) \equiv G_\rho(x; \varphi = \pi)$  and  $G^{(2)}(x; \tilde{\phi}) \equiv G_\rho(x; \varphi = [1 - (\rho \Sigma_{\pi N}/f_\pi^2 m_\pi^2)]\tilde{\phi})$ . These Green functions are not identical but their pole positions give the same effective mass,  $m_\pi^*(2)$  of eq.(9). On the other hand,

if we consider the Green function  $G^{(1)}(x; \phi) \equiv G_\rho(x; \varphi = \phi)$  governed by  $\mathcal{L}^{(1)}$ , with  $\phi$  being the field appearing in (1), the pole position will move to  $m_\pi^*(1)$  [eq.(4)], reflecting the change of the basic Lagrangian. Now, if one has a definite criterion to decide which effective Lagrangian,  $\mathcal{L}^{(1)}$  or  $\mathcal{L}^{(2)}$ , is superior, then one would know which effective mass to use,  $m_\pi^*(1)$  or  $m_\pi^*(2)$ . So, the crucial question is whether the formalisms so far developed allow us to decide which of  $\mathcal{L}^{(1)}$  and  $\mathcal{L}^{(2)}$  is a better choice.

From the ChPT point of view, one might assert that, to a given chiral order,  $\mathcal{L}^{(1)}$  is unique (modulo field transformations) and hence any other Lagrangians, including  $\mathcal{L}^{(2)}$ , that deviate therefrom within the same chiral order should be discarded. The issue, however, is more subtle. We have shown in (14) that  $\mathcal{L}^{(1)}$  and  $\mathcal{L}^{(2)}$  differ by terms of  $\mathcal{O}(\rho^2)$ . However, since  $\mathcal{L}^{(1)}$  is devoid of terms containing  $(\bar{N}N)^2$  like  $\pi^2(\bar{N}N)^2$  (which in the mean-field approximation would give contributions of  $\mathcal{O}(\rho^2)$ ), it goes beyond the accuracy of  $\mathcal{L}^{(1)}$  to discuss the difference of  $\mathcal{O}(\rho^2)$  between  $\mathcal{L}^{(1)}$  and  $\mathcal{L}^{(2)}$ . If  $\mathcal{L}^{(1)}$  were a fundamental Lagrangian, one might still be able to justify the absence of terms involving  $(\bar{N}N)^n$  ( $n \geq 2$ ) in (1) [26]. However,  $\mathcal{L}^{(1)}$  being an effective Lagrangian, one cannot a priori exclude from  $\mathcal{L}^{(1)}$  terms containing  $(\bar{N}N)^n$  ( $n \geq 2$ ) [28]. Therefore, there is no compelling reason to prefer  $\mathcal{L}^{(1)}$  to  $\mathcal{L}^{(2)}$ .

Meanwhile, from the current-algebra-plus-PCAC viewpoint, one might claim that  $\mathcal{L}^{(2)}$  is a “natural” choice, and that  $\mathcal{L}^{(1)}$  is an approximate Lagrangian obtained from  $\mathcal{L}^{(2)}$  by ignoring the  $\mathcal{O}(\rho^2)$  terms in eq. (14). However, this argument is subject to the same criticism as above; therefore  $\mathcal{L}^{(2)}$  cannot be considered as a better approximation than  $\mathcal{L}^{(1)}$ .

These observations make it clear that the true understanding of the difference between  $m_\pi^*(1)$  and  $m_\pi^*(2)$  requires a knowledge of terms of  $\mathcal{O}(\rho^2)$  in the effective Lagrangian itself. In other words, the discrepancy between  $m_\pi^*(1)$  and  $m_\pi^*(2)$  represents the effects of two (or more) -nucleon interaction terms which have not been addressed so far. This is a new type of matter effect. Usually, matter effects of  $\mathcal{O}(\rho^2)$  such as the Lorentz-Lorenz-Ericson-Ericson effect, the in-medium modifications of  $g_A$ ,  $m_N$  etc., are regarded as well-defined corrections to the linear-density approximation for ChPT. However, the above discussion shows that there exists a class of matter effects which arise from higher-order density terms in the effective Lagrangian, and whose form vary according to the extrapolating field. We re-emphasize that this extrapolating-field dependence does not affect  $m_\pi^*$  if no truncation is introduced to the chiral effective Lagrangian, and if  $G_\rho(x; \varphi)$  is treated exactly. It is only when an approximation is introduced either in  $\mathcal{L}$  or in the calculation of  $G_\rho(x; \varphi)$  that the resulting  $m_\pi^*$  becomes dependent on the interpolating field.

So far we concentrated on the sigma term in the pion sector. The situation is essentially the same for the kaon case. We now discuss the meaning of our argument in the light of the recent developments in the ChPT approach to kaon condensation [7,8]. The basic problems with the earlier ChPT calculations [1-5] were: (i) chiral-counting was not done consistently; (ii) the  $K$ - $N$  scattering amplitudes did not possess a correct energy dependence to reproduce the scattering data.

Regarding problem (i), a systematic ChPT calculation that respects chiral-order counting based on the heavy-fermion formalism has been carried out to the tree order by Brown, Lee, Rho and Thorsson [7], and to the one-loop order by Lee, Jung, Min and Rho [8, 29]. As far as the ordinary chiral counting in vacuum is concerned, these calculations are complete up to the stated chiral orders, but multiple-fermion terms do not appear in these calculations [30]. This gives the impression that we need not worry about the absence of multi-fermion terms in  $\mathcal{L}^{(1)}$ . However, because a finite-density system has an additional scale  $\rho$ , chiral counting in nuclear matter is not as firmly established as in vacuum. This caveat becomes particularly important in applying ChPT to systems of higher densities. Thus, a further study is needed to check whether the contributions of multiple-fermion terms are as suppressed as the ordinary chiral counting would indicate [31]. Until this point is settled, it seems unsafe to invoke the ordinary chiral counting to justify ignoring  $\mathcal{O}(\rho^2)$  terms that are responsible for the difference between  $\mathcal{L}^{(1)}$  and  $\mathcal{L}^{(2)}$  [32]. Even if one insists on the ordinary chiral counting, the inclusion of meson loops in the ChPT calculations requires, to the same chiral order, that at least two nucleon terms,  $\mathcal{O}(\rho^2)$  terms, should in general be taken into account in calculating the Green function in nuclear matter. In this sense also,  $\mathcal{O}(\rho^2)$  terms like  $\pi^2(\bar{N}N)^2$  need to be included in the Lagrangian itself.

As for problem (ii), Lee et al.[8] considered the energy dependence coming from the one loop diagrams and the resonance  $\Lambda^*(1405)$ , and were able to reproduce reasonably well the existing data on the s-wave  $K$ - $N$  scattering amplitude. The pronounced energy dependence in the s-wave  $\bar{K}$ - $N$  ( $I = 0$ ) scattering amplitude was reproduced by adjusting the parameters characterizing  $\Lambda^*$ . However, the limited precision of the present experimental data hinders an accurate test of the energy dependence due to the loop diagrams; this dependence should be most visible in the  $K$ - $N$  ( $I = 1$ ) channel. Now, since kaon condensation depends on the energy behavior of the  $K$ - $N$  amplitudes from threshold ( $\omega = m_K$ ) down towards  $\omega = 0$ , it is important to know whether this subthreshold energy behavior is reproduced accurately by the one-loop corrections. In the language of the empirical low-energy expansion,

$$T_{KN} = a + b(\omega^2 - m_K^2) + \mathcal{O}((\omega^2 - m_K^2)^2), \quad (15)$$

this means that the parameters in  $\mathcal{L}$  must account not only for the s-wave  $K$ - $N$  scattering length  $a$  but also for the s-wave effective range  $b$  [33]. This is at present a difficult task due to the paucity of experimental data. In the phenomenological approach of [11] this difficulty is reflected in the fact that the  $\Sigma_{KN}$  was treated as a parameter. Further experimental information on low-energy  $K$ - $N$  scattering as well as calculations that include  $\mathcal{O}(\rho^2)$  terms are required to make progress in this discussion.

Finally, there is a possibility of using astrophysical input to place constraints on the role of the multi-fermion terms in the effective chiral Lagrangian. According to Brown and Bethe [34], the kaon condensate, if it exists, should lead to the formation of “nuclear star” matter (instead of the neutron star matter) and the proliferation of pygmy blackholes. If observational support for this scenario becomes compelling enough, that may be construed as indirect evidence for the essential correctness of the effective Lagrangian so far used in the ChPT approach to kaon condensation.

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$$T_{\pi N}^{(2)} = \frac{\langle N|\bar{u}u + \bar{d}d|N\rangle}{\langle 0|\bar{u}u + \bar{d}d|0\rangle}(k^2 + (k')^2) + T'_{\pi N}|_{m_\pi=0}.$$
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